

## ACCESS NETWORK PLANNING: MINIMIZATION OF CONSTRUCTION COST

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### Abstract

*A fiber-optic cable-laying routing with a minimum cost is one of the most important issues of Access Network Planning. In this paper, the problem of routing between a local exchange (LE) and its remote units for an access network (AN) ring structure is considered. We formulate this problem as combinatorial-optimization one and use dynamic programming principle for its solution. Software calculation results demonstrate the capability of our approach to determine the minimum-cost route.*

**Keywords:** Access Network, Dynamic Programming, Bellman principle

### 1. INTRODUCTION

ANs undergo an essential evolution. There are some important aspects of this global process. The considerable growth of Internet traffic and the deployment of commercial on-line and multimedia services has led to dramatic increase in demand for data transmission capacity. The perspective AN should satisfy the following requirements:

- To support the broadband connections for Internet, Switched Multimegabit Data Services (SMDS), etc;
- To have high reliability;
- To use the network resources effectively;
- To provide (if necessary) services for terminal mobility.

The model of a perspective AN is depicted in Fig. 1.

The AN has the ring structure. It is based on a fiber-optic cable passing over a number of remote units [1,2]. The subscribers are connected to the LE and the remote units with individual subscriber lines. By a remote unit (RU) is meant a multiplexer, a remote subscriber unit, a base station (for wireless access) or a combination of these elements. A Digital Cross Connect and Add-Drop

Multiplexer of the transport network may play the role of the multiplexer [3]. Fiber-optic cables are widely used in modern ANs. There are several scenarios of using a fiber-optic cable: Fiber To The Apartment, Fiber To The Building, Fiber To The Curb, Fiber To The Home, Fiber To The Office, Fiber To The Remote, Fiber To The Zone and others [4,5]. Accordingly, the principles and tools of AN planning are different.

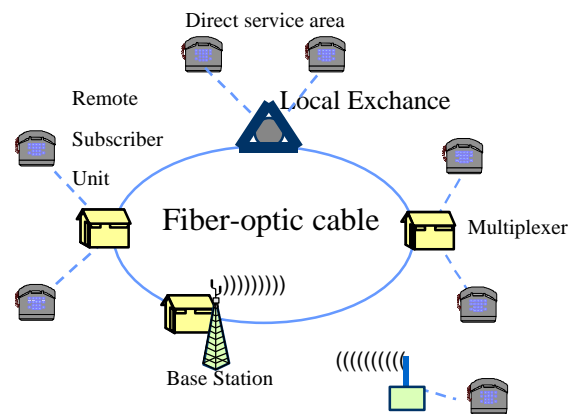


Figure 1. Model of a perspective AN

By the most conservative estimation, the cost of an AN construction is above 30% - 40% of telecommunication system construction total cost [6-8]. For these reasons, the elaboration of AN planning methods has become an important area of researches in field of telecommunications. The methods should minimize the expenses on this element of telecommunication system and satisfy the requirements mentioned above.

For development of such methods the following interdependent problems should be solved:

- Choice of a fiber-optic cable-laying route between a LE and its RUs for AN construction with minimum cost;
- Optimum accommodation of RUs equipment;

- Estimation of AN reliability parameters (for different AN structures);
- Choice of the AN structure using cost and reliability criteria.
- Choice of an optimal number of the RUs.

In this paper, the problem of cable-laying ring routing between a LE and its RUs is considered. The objective is to determine the minimum-cost ring route that satisfies the specified constraints. We pose this as a combinatorial-optimization problem of high complexity (NP-complete) and demonstrate the ability of dynamic programming principle to provide acceptable solution for it.

## 2. DEFINITION OF THE ROUTING PROBLEM

### Problem

It is given: the LE, the set of RUs, the set of links connecting these nodes with each other.

It is necessary: to find the shortest path (*on the cost optimum criterion*) that satisfies to the following constraints:

- To pass over the LE and all RUs,
- To pass over each RU only once,
- To begin and to end in the same point corresponding to the LE,
- To have no splits.

We shall number the nodes:

$$x \in \{1, 2, \dots, N\}.$$

The cost of the link between  $x_i$  and  $x_{i-1}$  nodes is  $(x_{i-1}, x_i)$ , where  $i$  denotes the place of a node on the ring route. The cost may consist of the following components

$$(x_{i-1}, x_i) = \alpha\rho + \beta g + \gamma f + v, \quad (1)$$

where  $\alpha$  - the cost of the fiber-optic cable length unit;  $\rho$  - the distance between  $x_i$  and  $x_{i-1}$  nodes;  $\beta$  - the complexity coefficient of the cable-laying;  $g$  - the cost of the cable-laying;  $\gamma$  - the complexity coefficient of installation works dependent on methods of the cable-laying;  $f$  - the installation works cost of the cable-laying;  $v$  - the additional expenses dependent on some specific local conditions.

The problem is to determine

$$\min \sum_{i=1}^N C(x_{i-1}, x_i) = \min_{\{x_2, \dots, x_{N+1}\}} J_{N+1}(x_1, x_2, \dots, x_N, x_{N+1}), \quad (2)$$

taking into consideration the above-stated constraints. Here,  $J_{N+1}(x_1, x_2, \dots, x_N, x_{N+1})$  is the path cost equal to the total cost of links in it.

## 3. APPROACH FOR DETERMINATION OF THE RING ROUTE

The known shortest-path algorithms do not suit for it because of the above-stated constraints. For many of such problems optimal solutions cannot be guaranteed [9]. We present the approach using the dynamic programming principle to solve the problem [10]. The dynamic programming principle has been successfully used for Network Planning [11,12].

Let's cut the ring, for example, in the node 1, which corresponds to the LE location. Then, it is necessary to find the shortest path beginning in the node  $x_1=1$  and coming to its end in the node  $x_{N+1}$ , as shown in Fig. 2. It is clear, that the node  $x_{N+1}$  coincides with the node  $x_1$ :

$$x_1 = x_{N+1} = 1.$$

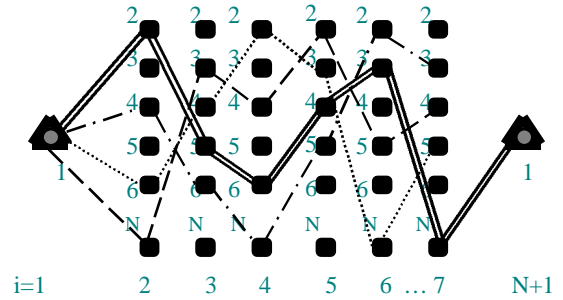


Figure 2. Illustration of the approach for determination of the ring route with the minimum cost

There are nodes from 2 to  $N$  on intervals  $i=2, 3, \dots, N$ .

The state of the system on  $i$ -step

$$x_i \in \{2, 3, \dots, N; i=2, 3, \dots, N\}.$$

The form of the objective function for the path with its length of  $i$ -links is given by

$$J_{i=2, \dots, (N+1)}(x_1, x_2, \dots, x_i) = J(x) = \begin{cases} \hat{J}(x_{i-1}) + C(x_{i-1}, x_i), & \text{if } x \cap \{\hat{x}_1, \dots, \hat{x}_{i-1}\} = \emptyset \\ \infty, & \text{if } x \cap \{\hat{x}_1, \dots, \hat{x}_{i-1}\} \neq \emptyset, \end{cases} \quad (3)$$

where  $\hat{J}(x_{i-1})$  - the survive path cost equal to the sum of link costs from the node 1 to the node  $x_{i-1}$ .

Using the Bellman principle [10] we have the system of  $N$  functional equations (for each value  $x_i$ ):

$$\tilde{J}(x) = \begin{cases} \min_{x \rightarrow x_{i-1}} \hat{J}(x_{i-1}) + C(x, x_{i-1}), & \text{if } x \cap \{\hat{x}_1 \dots \hat{x}_{i-1}\} = 0 \\ \infty, & \text{if } x \cap \{\hat{x}_1 \dots \hat{x}_{i-1}\} \neq 0 \end{cases} \quad (*)$$

Solving (4) we get a variant of the fiber-optic cable-laying ring route that has the nearly optimal cost. The restriction (\*) allows us to take into account the constraint (i) according to which the path can pass only once over each node. Unfortunately, there is no guarantee that optimal solutions will be found in all cases. However, imposing of this restriction will allow:

- To reduce the solution space up to  $\sim N^3$  (N - the number of nodes);
- To take into account the above-stated constraints;
- To get the variant of the ring route being close to the optimum.

It is unlikely, but the restriction (\*) may lead to some incorrectness of the result, in particular, when the number of nodes is considerable [9]. In practice, the number of RUs on the ring does not exceed 6 or 7. Nevertheless, it is necessary to repeat the above-stated procedure of the path quest by cutting the ring in other nodes:

$$x_i = x_{N+1} = 2, 3, \dots, N.$$

The minimum-cost path should be chosen for cable-laying routing. These operations shall increase slightly the number of calculations (up to  $\sim N^4$ ) but it will make sure that the variant of route determined earlier is right.

#### 4. AN EXAMPLE OF THE DETERMINATION OF THE ROUTE FOR SEVEN-NODE ACCESS NETWORK

In this section we demonstrate the ability of our approach to find the minimum-cost route by the following example.

It is given:

- 1) the LE and 6 RUs (Fig. 3).
- 2) the cost matrix of the links connecting the nodes

$$|\tilde{N}| = \begin{vmatrix} 0 & 0.333 & 0.048 & 0.952 & 0.905 & 0.714 & 0.476 \\ 0.333 & 0 & 0.381 & 1 & 0.857 & 0.286 & 0.429 \\ 0.048 & 0.381 & 0 & 0.524 & 0.095 & 0.762 & 0.571 \\ 0.952 & 1 & 0.524 & 0 & 0.809 & 0.238 & 0.190 \\ 0.905 & 0.857 & 0.095 & 0.809 & 0 & 0.619 & 0.143 \\ 0.714 & 0.286 & 0.762 & 0.238 & 0.619 & 0 & 0.666 \\ 0.476 & 0.429 & 0.571 & 0.190 & 0.143 & 0.666 & 0 \end{vmatrix}$$

It is necessary: to find the shortest path passing over the LE and its RUs on a ring.

The approach described in Section III was used for software calculations, which gave the shortest path shown in Fig.3.

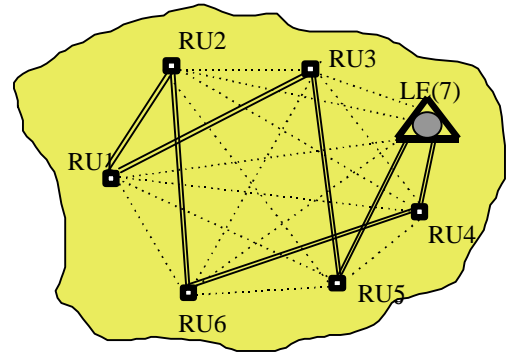
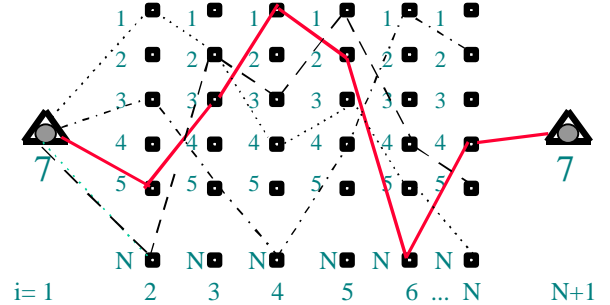


Figure 3. An example of the determination of the ring route with the minimum cost for seven-node Access Network

The cost of the path equals 1.333. For comparison, the value of the cost bottom boundary of path that satisfies the constraints (i) and (ii) comes to 1.047. The difference between these values is less than 28%. The cost bottom boundary was expressed as follows

$$\tilde{N}_{opt} \geq \sum_{i=1}^N \min_j C(x_i x_j), i \neq j, j = \overline{1, N}, \quad (5)$$

where  $C(x_i x_j)$  - the cost of the link between  $x_i$  and  $x_j$  nodes.

If we cut the ring in other nodes and repeat all calculations, the minimum-cost path passes by the same route. Consequently, this variant of the cable-laying ring route can be considered acceptable for the AN planning. In contrast, the cost of the radial fiber-optic cable-laying route from the LE to its RUs is 2.475. It is of 86 % more than the cost of the ring route.

Such software calculations were run for different number of nodes. The calculations have

been able to produce appropriate results in all runs. The fact that the most of results were very close to the optimum value in such a large fraction of the cases studied demonstrates the robustness of our approach, and allows to suggest that it may perform well in considerably larger examples as well.

## 5. CONCLUSIONS

In this paper, we have formulated a version of routing problem in an AN ring structure as a combinatorial-optimization problem, and used dynamic programming to solve it. Our approach enables to get the fiber-optic cable-laying route between the LE and its RUs being close to optimum. The approach allows to reduce the expenses on fiber-optic cable and its laying when constructing an AN. It also allows to considerable reduce the solution space up to  $\sim N^4$ , where  $N$  - the number of nodes on the ring. Software calculation results demonstrate the effectiveness of the approach for the minimization of the ring route cost. The submitted approach has another advantage. The non-ring route close to the bottom boundary of cost (5) can arise on intermediate stages. It enables to choose different structures of an AN construction. It should be emphasized that at the final choice of the AN structure, there is a need to take into account the reliability parameters and the complexity of the network management, as well.

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